Electronic synchronization of gain-switched laser diode seeded fiber amplifiers
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ABSTRACT
We present a hybrid laser system based on all-fiber amplification of a gain-switched laser diode. The diode emits low energy pulses with several tens of picoseconds pulse duration at a wavelength of 1550 nm and a repetition rate of 1 MHz. The three-stage fiber amplifier reaches an overall gain of 55 dB boosting the pulse energy to 0.48 μJ. Much care is taken to preserve an almost bandwidth limited pulse with a spectral width of 0.1 nm and negligible spectral broadening due to nonlinearities. The laser system has been designed such that two or more can be electronically synchronized with the aim to combine them for exploring frequency mixing scenarios. Here, we report first cross-correlation measurements of two synchronized laser systems and present a method to characterize the relative timing-jitter.

1. INTRODUCTION
Gain-switched semiconductor laser diodes are very attractive sources of picosecond pulses\(^1\)–\(^3\) because they are compact, stable, and provide tunable repetition rates from pulse-on-demand up to hundreds of megahertz. They are a good alternative to picosecond mode-locked lasers, which are more expensive, less robust, and require complex long-cavity designs or pulse pickers to reach repetition rates below 10 MHz. Gain-switched laser diodes, however, have a few drawbacks. First, their pulse energy is typically only a few tens of picojoules and, consequently, the pulses need to be amplified by several tens of dBs in order to reach energy levels nowadays available from mode-locked lasers. Second, gain-switched laser diodes exhibit higher timing-jitter. Third, the output pulses may be chirped because of the gain dynamics.\(^4\)–\(^6\) Several research activities aim for a better stabilization and an improved control over the gain dynamics.\(^7\)–\(^10\) One of the most attractive features of gain-switched laser diodes is that, being electrically pumped with a pulsed current, the same electrical signal can be used to trigger more than one diode at a time, facilitating the synchronization of multiple diodes. Electrical synchronization presents a simple and cheap way to realize multi-wavelength sources. Several synchronized lasers can then be used for nonlinear frequency mixing, time-resolved pump-probe spectroscopy, and all other measurement techniques that require synchronized laser sources in the picosecond regime.

In this paper we exemplarily present the design of an Er-fiber amplifier for a gain-switched laser diode emitting 40 ps pulses at 1 MHz repetition rate and a wavelength of 1550 nm. The average seed power is only 2.15 μW and several amplification stages and filtering techniques are required to guarantee high fidelity amplification, suppressing amplified spontaneous emission (ASE) and thereby obtaining a good signal-to-ASE ratio. Much care is also taken to avoid spectral broadening due to strong saturation and nonlinear effects in the fibers, which are typically observed when working in highly-saturated regimes. We use similar designs to build two additional laser systems at 1064 nm and at 1540 nm and we briefly review their performances. In the second section of this paper we explain how we characterize the fluctuations of pulse energy, peak power and duration of each individual laser system using a method proposed in references\(^11\)–\(^12\). In the third section, we report the design of an experiment on sum-frequency generation (SFG) in a nonlinear periodically poled LiNbO\(_3\) crystal using two electronically synchronized laser systems. Here, the experimental setup is predominantly used as a diagnostic tool to estimate the relative timing-jitter between the two laser systems.

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2. MULTI-STAGE FIBER AMPLIFIER

In this section we describe in detail a laser system operating at 1550 nm and only summarize the performance of the other systems, since their design is very similar. The gain-switched laser diode is a distributed-feedback laser (DFB-LD) at 1550 nm providing 40 ps long pulses with a pulse energy of 2.15 pJ at a repetition rate of 1 MHz (average power 2.15 μW). The DFB-LD is provided by Advanced Laser Diode Systems. The spectrum and temporal intensity profile are shown in Fig. 1. The pulses show an almost perfect Gaussian shaped profile, but the spectrum is relatively broad, which is a typical scenario for DFB gain-switched laser diodes.

The aim of the fiber amplifier is to boost the pulse energy without any spectral or temporal broadening, possibly even improving the spectral profile, providing at the same time a good signal-to-ASE ratio. Given the very low average power of the DFB-LD, the first stage of the amplifier is crucial for the overall amplifier performance. The first thing to consider is the fact that the amount of ASE is inversely proportional to the signal power density. For this reason, when dealing with very low-power seed pulses, it is advantageous to use an active fiber with a very small core. On the other side, during amplification, the seed power increases, and in small-core fibers it can easily reach the threshold for nonlinear effects to occur, hence limiting the optimal gain of the amplifier well below the maximum gain of the fiber. Ideally, it would be useful to work with fibers whose core size increases with fiber length. An alternative scheme is to use a multi-stage amplifier where the fiber core increases from stage to stage. Following this scheme, our fiber amplifier system is based on a three-stage setup and consists of two core-pumped Er-doped fiber preamplifiers followed by a commercial ErYb-doped double-clad soft-glass fiber amplifier. The setup is depicted in Fig. 2. Each Er-doped fiber amplifier (EDFA) stage is pumped with a semiconductor laser diodes at 976 nm. Spectral filtering between stages is used to remove most of the ASE. The Er-doped fibers and all other fiber components are polarization-maintaining (PM).

The first stage of the preamplifier uses a PM Er-doped fiber with a mode-field diameter of about 5μm. Despite the fiber can provide more than 35 dB gain, as shown in Fig. 3b, we limit the amplification to the optimal value of 16.5 dB. Here, neither nonlinear nor saturation effects occur and the competing ASE still remains at reasonably low levels (signal-to-ASE ratio is 0.03 dB).
After the second stage of the preamplifier we reach a total output power of 14 mW (14 nJ pulse energy), which corresponds to a gain of 21.7 dB. Using another inline fiber filter we achieve suppressing the ASE to a 26 dB signal-to-ASE ratio. The maximum gain in this second stage is mainly limited by the onset of nonlinear effects and by the pump power. The maximum output power we can extract at full pump power (500 mW) is 70 mW (70 nJ pulse energy), but above the optimal signal power of 14 mW spectral broadening starts to occur due to nonlinear effects in the fiber.

Finally, the boost amplification stage is based on a commercial ErYb-doped double-clad soft-glass large-
mode-area fiber amplifier module Blizzard-1.5μm from Polar Laser Laboratories. If seeded with 14 mW from the previous stage, the system can deliver a stable signal of 0.485 W average power, which corresponds to pulse energy of 0.485 μJ and a peak power of 11.4 kW. The maximum average power which can be extracted from this amplifier is 1 W (1 μJ pulse energy), as shown in Fig. 5b. Unfortunately, the system is not stable at these power levels because, due to the low input signal power, strong onset of ASE can lead to Q-switching dynamics which can damage the amplifier.

The signal-to-ASE ratio at the output of the boost amplifier is 14 dB (the signal power corresponding to 96% of the total output power). The output spectrum, measured with a resolution of 0.02 nm, is reported in logarithmic scale in Fig. 5a. The signal, whose spectrum has a FWHM of only 0.1 nm, can be again clearly distinguished from the ASE contribution, coming from both previous stages, as well as from the last amplification stage.

![Figure 5: Output of the boost amplifier. (a) Spectrum, (b) output power and gain.](image)

Such a good signal-to-ASE ratio for an amplified gain-switched laser diode at 1 MHz has not, to our knowledge, been reported yet, and it is comparable only to the most recent results obtained in Yb-doped fiber amplifiers. But the main achievement of this system is that we succeeded to amplify the seed by several dBs, providing also a very narrow spectrum and almost transform-limited pulses. In Fig. 6a we compare the output spectrum of the boost amplifier to the spectrum of the seed. Spurious spectral components of the gain switched laser diode have been filtered out in the preamplifier and the spectrum presents an almost Gaussian profile with 0.1 nm FWHM spectral width.

![Figure 6: Output of the boost amplifier. (a) spectrum in linear scale, (b) temporal pulse profile.](image)

Amplification occurs without significant temporal distortions, as can be see in Fig. 6b. The time-bandwidth...
product corresponds to 0.57, approaching the theoretical transform-limited value for a Gaussian pulse (0.44). The very low noise figure and the narrow spectral width make this system an extremely interesting picosecond laser source for nonlinear frequency mixing in crystals. In fact, in order to have efficient nonlinear frequency conversion at these power levels the spectrum must be well within the acceptance bandwidth of the nonlinear crystal. Also for this reason, special care has been taken to avoid spectral broadening due to the onset of saturation and nonlinear effects in the fibers. The high-fidelity amplification and pulse quality of this laser system are extremely promising, especially because further power scaling can be achieved in large-mode Er-doped fibers without pulse breakup, as it has been demonstrated in the ps regime.

Using a similar amplifier design as described in this section, we developed other two laser systems at 1064 nm and at 1540 nm. Each laser system is seeded with a gain-switched DFB-LD and amplified with a two-stage fiber preamplifier. The amplifier at 1064 nm is based on Ytterbium-doped fiber amplifier technology (YDFA). In table 1 below we compare the main characteristics of the preamplifiers of the three laser systems.

<table>
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<th>1064 nm</th>
<th>1540 nm</th>
<th>1550 nm</th>
</tr>
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<tr>
<td>Pulse length $\tau_{FWHM}$</td>
<td>39 ps</td>
<td>27 ps</td>
<td>40 ps</td>
</tr>
<tr>
<td>Optimal pulse energy</td>
<td>10 nJ</td>
<td>14 nJ</td>
<td>15 nJ</td>
</tr>
<tr>
<td>Optimal peak power</td>
<td>241 W</td>
<td>487 W</td>
<td>352 W</td>
</tr>
<tr>
<td>Max pulse energy</td>
<td>50 nJ</td>
<td>20 nJ</td>
<td>70 nJ</td>
</tr>
<tr>
<td>Max peak power</td>
<td>1.2 kW</td>
<td>0.7 kW</td>
<td>1.6 kW</td>
</tr>
</tbody>
</table>

Table 1: Properties of the three laser systems.

Optimal pulse energies and peak powers refer to amplification without spectral distortions. The maximum pulse energies and peak powers report the maximum values we can extract from the systems. Usually they are well above the nonlinear power threshold of the fibers. While the maximum pulse energy at 1064 nm and 1550 nm is limited by the maximum pump power, the maximum energy of the system at 1540 nm is limited by saturation effects in the amplifier.

3. STATISTICS OF A SINGLE LASER SYSTEM

In order to perform and understand the nonlinear frequency mixing experiments it is important to characterize the stability of the pulse energy as well as the fluctuations of pulse duration and peak power of each laser system involved. For that, we adopt a method based on autocorrelation techniques. The method, proposed in 1975 and whose more detailed description is reported in reference, is very simple and relates the fluctuations of the pulse energy and its second harmonic to the fluctuations of the peak power and pulse duration. We briefly describe it in Sec. 3.1. The experimental setup is quite simple, i.e. each laser is sent through a nonlinear crystal, in our case a periodically-poled LiNbO$_3$ crystal (PPLN), and both the fundamental wave (FW) and its second-harmonic (SH) are detected simultaneously with a ‘slow’ photodiode. The rising time of the photodiode is longer than the pulse duration, but its response time is shorter than the duty-cycle of the laser (1 $\mu$s for 1 MHz repetition rate). In this way the charge accumulated by the photodiode is proportional to the pulse energy and we make sure that only one pulse at a time is measured. The electrical signal from the photodiode is analyzed by a digital oscilloscope (Tektronix DPO 7254) with a temporal resolution of 50 ps. Each time scan contains about one hundred pulses and an example is shown in Fig. 7a.

From such a trace we can calculate the average pulse energy $\langle E \rangle$ and its standard deviation $\sigma_E$. In Fig. 7b the pulse energy distribution for the laser system at 1540 nm and its second harmonic wave are shown with respect to the average value. The distribution of energy values is well described by a Gaussian distribution whose width corresponds to the standard deviation of the pulse energy and gives a measure of the fluctuations of the pulse energies.

3.1 Theoretical model

In the following we describe the theoretical model used to extract the statistics of the pulse peak power and the pulse duration from a measurement of the pulse energies of the fundamental and the second harmonic wave. We
Figure 7: (a) A sequence of six pulses together with the average pulse energy $\langle E \rangle = \bar{E}$ and its standard deviation $\sigma_E$ indicated by the dashed horizontal lines. (b) Distribution of the pulse energy around the average value for both the fundamental wave at 1540 nm (orange) and its second harmonic at 770 nm (blue).

Assume that the pulses are Gaussian in time. Given a pulse of temporal duration $\tau$ and peak power $P_0$, its power profile can be written as:

$$P(t) = P_0 e^{-t^2/\tau^2}$$  \hspace{1cm} (1)

The energy $E_{FW}$ of a single fundamental pulse is related to its power $P(t)$ through the relation

$$E_{FW} = \int_{t_1}^{t_2} P(t) dt = \sqrt{\pi} P_0 \tau$$  \hspace{1cm} (2)

provided that $t_2 - t_1 \gg 2\tau$ and $t_2 - t_1 < 1/R$, where $R$ is the repetition rate of the laser. If we now consider the second harmonic generation in the undepleted pump approximation, we obtain the pulse energy of the second harmonic signal

$$E_{SH} = \kappa \int_{t_1}^{t_2} P^2(t) dt = \kappa \sqrt{\pi} \frac{P_0^2}{2} \tau = \kappa \frac{P_0}{\sqrt{2}} E_{FW}$$  \hspace{1cm} (3)

where the coefficient $\kappa$ takes into account several parameters which are considered independent of the power profile, like the beam area, the nonlinear susceptibility, and the length of the nonlinear crystal.\textsuperscript{11,23} It is clear from Eq. (2) and Eq. (3) that the measurement of the pulse energy of both the fundamental wave and its second harmonic provides information on the peak power and the pulse duration of the pulse. In the following we describe how fluctuations of the peak power $P_0$ and temporal width $\tau$ can be retrieved from the measurement of the fluctuations of the pulse energies $E_{FW}$ and $E_{SH}$.

We consider a realistic situation where the peak power $P_0$ and pulse duration $\tau$ are fluctuating from shot to shot, but their fluctuations are small compared to the average value, so that $P_0$ and $\tau$ can be expanded in a Taylor series up to the first order around their average value

$$P_0 = \bar{P} + \Delta P = \bar{P}(1 + \xi_P)$$  \hspace{1cm} (4)

$$\tau = \bar{\tau} + \Delta \tau = \bar{\tau}(1 + \xi_\tau)$$  \hspace{1cm} (5)

*In this notation the FWHM pulse duration corresponds to $\tau_{FWHM} = 2\sqrt{\ln(2)}\tau$.\textsuperscript{11}
where $\bar{P}$ and $\bar{\tau}$ are the average values of the peak power and the pulse duration, $\Delta P$ and $\Delta \tau$ and $\xi_P = \Delta P / \bar{P}$ and $\xi_{\tau} = \Delta \tau / \bar{\tau}$ are the absolute and the relative fluctuations, respectively. By introducing the absolute and relative fluctuations of the pulse energies $\xi_{FW} = \Delta E_{FW} / \bar{E}_{FW}$ and $\xi_{SH} = \Delta E_{SH} / \bar{E}_{SH}$ and substituting Eqs. 4 to 5 into Eqs. 2 to 3 we derive the relative fluctuations of the pulse energies as a function of the relative fluctuations of the peak power and the pulse duration

$$\begin{align*}
\xi_{FW} &= \xi_P + \xi_{\tau} \\
\xi_{SH} &= 2\xi_P + \xi_{\tau}
\end{align*}$$

(6)

Using Eqs. 6 we calculate the variance of $E_{FW}$ and $E_{SH}$ as a function of the variance of the peak power and the pulse duration

$$\begin{align*}
\sigma_{FW}^2 &= \sigma_P^2 + \sigma_{\tau}^2 + 2\sigma_P \sigma_{\tau} \\
\sigma_{SH}^2 &= 4\sigma_P^2 + \sigma_{\tau}^2 + 4\sigma_P \sigma_{\tau} \\
\sigma_{FW,SH}^2 &= 2\sigma_P^2 + \sigma_{\tau}^2 + 3\sigma_P \sigma_{\tau}
\end{align*}$$

(7)

where $\sigma_{FW}^2 = \langle \xi_{FW}^2 \rangle$, $\sigma_{SH}^2 = \langle \xi_{SH}^2 \rangle$, $\sigma_{FW,SH}^2 = \langle \xi_{FW} \xi_{SH} \rangle$, $\sigma_P^2 = \langle \xi_P^2 \rangle$, $\sigma_{\tau}^2 = \langle \xi_{\tau}^2 \rangle$ and $\sigma_{P,\tau} = \langle \xi_P \xi_{\tau} \rangle$ and the brackets $\langle \rangle$ indicate the ensemble average. Eqs. (7) represent a linear system of coupled equations that can be inverted to

$$\begin{pmatrix}
\sigma_P^2 \\
\sigma_{\tau}^2 \\
\sigma_{P,\tau}
\end{pmatrix} =
\begin{pmatrix}
1 & 1 & -2 \\
4 & 1 & -4 \\
-2 & -1 & 3
\end{pmatrix}
\begin{pmatrix}
\sigma_{FW}^2 \\
\sigma_{SH}^2 \\
\sigma_{FW,SH}^2
\end{pmatrix}$$

(8)

That is, by measuring the fluctuations of the pulse energy of both the fundamental wave and its second harmonic we can retrieve the fluctuations of the peak power and pulse duration of the laser.

### 3.2 Experimental results

Using the method described in Sec.3.1 we measure the fluctuations of the pulse energy, peak power, and pulse duration of each laser system. All statistical analysis is based on an ensemble of 116 pulses. For each laser system, we measure the average pulse energy of the fundamental wave $\bar{E}_{FW}$, the relative energy fluctuations $\xi_{FW} = \Delta E_{FW} / \bar{E}_{FW}$, the variance $\sigma_{FW}^2 = \langle \xi_{FW}^2 \rangle$ and the standard deviation $\sigma_{FW} = \sqrt{\sigma_{FW}^2}$. We can also calculate the correlation coefficient $K_{FW,SH} = \sigma_{FW,SH} / (\sigma_{FW} \sigma_{SH})$ which measures the correlation between the FW and the SH pulse energies.

For the system at 1550 nm we measure $\sigma_{FW1} = 0.04$, $\sigma_{SH1} = 0.12$, and $K_{FW1,SH1} = 0.68$. The fluctuations of the FW pulse energy are very low, only 4% of the average pulse energy. Conversely, the fluctuations of the SH pulse energy are larger, i.e. 12% of the average SH pulse energy. We deduce that the FW pulses have almost constant energy, but the pulse duration shows significantly higher fluctuations. The correlation between the FW and SH pulse energies amounts to 68%. Its value is positive as expected (if the FW pulse energy increases, the SH pulse energy must increase as well), and it does not approach 100%, showing that there is an other source of SH pulse energy fluctuations, which is independent of the FW pulse energy. If we now substitute the measured quantities in Eq. 8 to estimate the fluctuations of the pulse duration and peak power we obtain $\sigma_{\tau_1} = 0.1$, $\sigma_{P_1} = 0.09$, and $K_{P_1,\tau_1} = -0.92$. The pulse duration and peak power amount almost to the same value, which was expected since, with the pulse energy almost constant, the pulse peak power fluctuations are mainly due to fluctuations in the pulse duration. The correlation coefficient $K_{P_1,\tau_1}$ is negative as expected (at constant pulse energy, if the pulse duration is increasing, the pulse peak power must decrease), and its absolute value is approaching unity, which shows that the pulse duration and the peak power are strongly correlated.

For the second laser system at 1540 nm we measure $\sigma_{FW2} = 0.02$, $\sigma_{SH2} = 0.13$, and $K_{FW2,SH2} = -0.04$. The fluctuations of the pulse energy of the fundamental wave are even smaller than for the previous laser system,
the pulse energy fluctuating by only 2%. The fluctuations of the SH pulse energy are slightly higher though. The correlation between the pulse energies is negligible and, given its very small value, its sign is basically meaningless. This might be surprising, considered the SH conversion process, from which we would expect the SH and the FW to be strongly correlated. But we explain this result as follows. If we retrieve the fluctuations of the peak power and the pulse duration using Eq. 8 we obtain \( \sigma_P^2 = 0.14, \sigma_\tau^2 = 0.14, \) and \( K_{P,\tau} = -0.99. \) In this case, the pulse duration and the peak power fluctuate exactly by the same amount and they are perfectly correlated. This means that the fluctuations of the peak power only depend on the fluctuations of the pulse duration, hence they do show any correlation with the (negligible) fluctuations of the FW pulse energy.

For the third laser system at 1064 nm we measure \( \sigma_{FW3} = 0.03, \sigma_{SH3} = 0.19, \) and \( K_{FW3,SH3} = 0.19. \) From these measurements we retrieve the fluctuations of the pulse peak power and duration, and their correlation, which are respectively \( \sigma_P^3 = 0.18, \sigma_\tau^3 = 0.18, \) and \( K_{P,\tau}^3 = -0.98. \)

Table 2 summarizes the average pulse duration and the relative and the absolute fluctuations for all three laser systems. We report here the FWHM pulse duration \( \bar{\tau}_{FWHM}. \) The fluctuations of pulse duration can be expressed in absolute value using the relation \( \Delta \tau = \sigma_\tau \bar{\tau}_{FWHM}. \) The fluctuations in pulse energy are very small compared to those numbers and, thus, they are not shown.

<table>
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<th>1550 nm</th>
<th>1540 nm</th>
<th>1064 nm</th>
</tr>
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<tbody>
<tr>
<td>( \bar{\tau} )</td>
<td>40 ps</td>
<td>27 ps</td>
<td>39 ps</td>
</tr>
<tr>
<td>( \sigma_\tau )</td>
<td>0.09</td>
<td>0.14</td>
<td>0.18</td>
</tr>
<tr>
<td>( \Delta \tau )</td>
<td>3.6 ps</td>
<td>3.8 ps</td>
<td>7.0 ps</td>
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Table 2: Statistics of all three laser systems.

4. SYNCHRONIZATION AND NONLINEAR FREQUENCY MIXING

The final section of this contribution analyzes two electronically synchronized laser systems, specifically their arrival time jitter. To extract quantitative numbers we use an extension of the method described in the previous section, i.e. the output of the two synchronized laser systems is sent to a sum-frequency generation crystal and the generated nonlinear signals as well as the fundamental signals are analyzed. A schematic of the setup is shown in Fig. 8.

![Figure 8: Scheme of the sum-frequency generation experiment. \( \lambda/2 \): half-wave plate; PD: photodiode; PPLN: periodically-poled LiNbO\(_3\) crystal; f: focal length.](image-url)
Two DFB-LDs, at 1540 nm and 1064 nm, are electronically synchronized with an RF signal and separately amplified in a two-stage fiber amplifier. A mechanical delay stage allows introducing a well-defined time delay between the two laser systems. Two waveplates adjust the polarizations for optimal frequency conversion. The nonlinear crystal is a periodically-poled LiNbO$_3$ crystal (Covesion$^{28}$). It is a 20-cm multi-period crystal, whose poling periods are 11.12 $\mu$m, 11.17 $\mu$m, and 11.22 $\mu$m, respectively. The phase-matching condition for 1540 nm and 1064 nm requires a period of 11.22 $\mu$m at a temperature of 205°C. After the crystal a prism is used to separate the different frequency components exiting the crystal and the different pulse energies are measured with suitable photodiodes and recorded with a fast DPO.

4.1 Theoretical model

It is the aim of this section to derive a link between the statistical analysis of the cross-correlation signal and the arrival time jitter between two synchronized laser systems. We denote the two fundamental waves with $E_1$ and $E_2$ and their respective second harmonics with $SH_1$ and $SH_2$. Again, we assume Gaussian pulses with a temporal width of $\tau_1$ and $\tau_2$ and a peak power of $P_1$ and $P_2$. With that we can write the cross-correlation signal as a function of the time delay $t^*$ between the two lasers

$$E_{SF} = k' \int I_1(t)I_2(t + t^*)dt$$

$$= k'\sqrt{\pi} \frac{P_1(\tau_1)P_2(\tau_2)}{\sqrt{\tau_1^2 + \tau_2^2}} e^{-t^2}$$

with the normalized time delay $T = t^*/\sqrt{\tau_1^2 + \tau_2^2}$. To evaluate the fluctuations of $t^*$, i.e. the arrival time jitter, we follow the same approach described in Sec. 3, but, in contrast to the previous model, we now have five variables that need to be considered. The fluctuations of the energy of the sum-frequency signal become

$$\Delta E_{SF} = \left| \frac{\partial E_{SF}}{\partial T} \right| \Delta T + \left| \frac{\partial E_{SF}}{\partial P_1} \right| \Delta P_1 + \left| \frac{\partial E_{SF}}{\partial P_2} \right| \Delta P_2 + \left| \frac{\partial E_{SF}}{\partial \tau_1} \right| \Delta \tau_1 + \left| \frac{\partial E_{SF}}{\partial \tau_2} \right| \Delta \tau_2.$$  (11)

where $\Delta T = \Delta t^*/\sqrt{\tau_1^2 + \tau_2^2}$. Eq. 11 shows that, by measuring the energy fluctuations $\Delta E_{SF}$, we can estimate the fluctuations of the normalized time delay $\Delta T$. By calculating the derivatives and introducing the relative errors, i.e. $\xi_{SF} = \Delta E_{SF}/E_{SF} \ldots$, we can rewrite Eq. (11) as

$$\xi_{SF} = \Gamma_T \xi_T + \xi_{P_1} + \xi_{P_2} + \Gamma_{\tau_1} \xi_{\tau_1} + \Gamma_{\tau_2} \xi_{\tau_2}.$$  (12)

with

$$\Gamma_1 = \frac{\tau_2^2 + 2\tau_1^2 T^2}{\tau_1^2 + \tau_2^2}$$  (13)

$$\Gamma_2 = \frac{\tau_1^2 + 2\tau_2^2 T^2}{\tau_1^2 + \tau_2^2}$$  (14)

$$\Gamma_T = 2T$$  (15)

and $\xi_T = \Delta T$. Together with those we can now define again all variances and covariances between the pulse energies and pulse parameters and write a system of fifteen linear equations for all variances and covariances. This system of equations can be decoupled into five independent subsystems. The first two subsystems are independent of the cross-correlation measurements and they can be solved using only the autocorrelation measurements.
The standard deviations $\sigma_{P_1}$, $\sigma_{P_2}$, $\sigma_{\tau_1}$, and $\sigma_{\tau_2}$ can then be determined through the formalism derived in the previous section where their values have also been reported. The third subsystem of equations is also independent of the cross-correlation measurement and allows to determine the correlations between the two lasers. Its solution provides the covariances $\sigma_{P_1, P_2}$, $\sigma_{P_1, \tau_1}$, $\sigma_{P_2, \tau_1}$ and $\sigma_{\tau_1, \tau_2}$ as function of $\sigma_{FW1,FW2}$, $\sigma_{FW1,SH2}$, $\sigma_{FW2,SH1}$ and $\sigma_{SH1,SH2}$. The fourth subsystem contains the correlations between the cross-correlation signal and the two fundamental waves. Its solution provides the covariances $\sigma_{T,P_1}$, $\sigma_{T,P_2}$, $\sigma_{T,\tau_1}$ and $\sigma_{T,\tau_2}$. Finally the fifth subsystem, which contains only the variance term $\sigma^2_T$, allows to retrieve the arrival time jitter between the two lasers, i.e. $\sigma_T = \sqrt{\langle \xi^2_T \rangle}$.

$$\Gamma^2_T \sigma^2_T = \sigma^2_{SF} - \begin{bmatrix} 1 & \Gamma_1^2 & 2 \Gamma_2 \\ \sigma^2_{P_1} & \sigma_{\tau_1}^2 & \sigma_{P_1,\tau_1} \\ \sigma_{P_1,\tau_1} & \sigma_{\tau_1}^2 & \sigma_{P_2,\tau_1} \\ \sigma_{P_2,\tau_1} & \sigma_{\tau_1}^2 & \sigma_{\tau_2}^2 \end{bmatrix}^{-1} - \begin{bmatrix} 1 & \Gamma_1^2 & 2 \Gamma_2 \\ \sigma^2_{P_1} & \sigma_{\tau_1}^2 & \sigma_{P_1,\tau_1} \\ \sigma_{P_1,\tau_1} & \sigma_{\tau_1}^2 & \sigma_{P_2,\tau_1} \\ \sigma_{P_2,\tau_1} & \sigma_{\tau_1}^2 & \sigma_{\tau_2}^2 \end{bmatrix}^{-1} \begin{bmatrix} \sigma_{T,P_1} \\ \sigma_{T,P_2} \\ \sigma_{T,\tau_1} \\ \sigma_{T,\tau_2} \end{bmatrix}$$

(16)

## Table 3: Summary of the measured as well as retrieved variances and covariances.

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<th>Measured values</th>
<th>Retrieved values</th>
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<td>$\sigma^2_{P_1} = \langle \xi^2_{P_1} \rangle$</td>
</tr>
<tr>
<td>$\sigma^2_{SH1} = \langle \xi^2_{SH1} \rangle$</td>
<td>$\sigma^2_{\tau_1} = \langle \xi^2_{\tau_1} \rangle$</td>
</tr>
<tr>
<td>$\sigma_{FW1,SH1} = \langle \xi_{FW1}\xi_{SH1} \rangle$</td>
<td>$\sigma_{P_1,\tau_1} = \langle \xi_{P_1}\xi_{\tau_1} \rangle$</td>
</tr>
<tr>
<td>$\sigma^2_{FW2} = \langle \xi^2_{FW2} \rangle$</td>
<td>$\sigma^2_{P_2} = \langle \xi^2_{P_2} \rangle$</td>
</tr>
<tr>
<td>$\sigma^2_{SH2} = \langle \xi^2_{SH2} \rangle$</td>
<td>$\sigma^2_{\tau_2} = \langle \xi^2_{\tau_2} \rangle$</td>
</tr>
<tr>
<td>$\sigma_{FW1,FW2} = \langle \xi_{FW1}\xi_{FW2} \rangle$</td>
<td>$\sigma_{P_1,P_2} = \langle \xi_{P_1}\xi_{P_2} \rangle$</td>
</tr>
<tr>
<td>$\sigma_{FW1,SH2} = \langle \xi_{FW1}\xi_{SH2} \rangle$</td>
<td>$\sigma_{P_1,\tau_2} = \langle \xi_{P_1}\xi_{\tau_2} \rangle$</td>
</tr>
<tr>
<td>$\sigma_{FW2,SH1} = \langle \xi_{FW2}\xi_{SH1} \rangle$</td>
<td>$\sigma_{P_2,\tau_1} = \langle \xi_{P_2}\xi_{\tau_1} \rangle$</td>
</tr>
<tr>
<td>$\sigma_{SH1,SH2} = \langle \xi_{SH1}\xi_{SH2} \rangle$</td>
<td>$\sigma_{\tau_1,\tau_2} = \langle \xi_{\tau_1}\xi_{\tau_2} \rangle$</td>
</tr>
<tr>
<td>$\sigma^2_{SF} = \langle \xi^2_{SF} \rangle$</td>
<td>$\sigma^2_{T} = \langle \xi^2_{T} \rangle$</td>
</tr>
</tbody>
</table>

### 4.2 Experimental results

In this section we report the experimental results that allow estimating the timing jitter between the two synchronized lasers. We consider Eq. 16 and notice that the first term on the right-hand side $\sigma^2_{SF}$ is the variance of the energy of the sum-frequency signal. This is experimentally measured in the same way as the pulse energy of the fundamental waves (see Sec. 3). The second and third term on the right-hand side are known and their values have been reported in Sec. 3. The first term of the second line of the right-hand side represents the correlations between the parameters of the two lasers. It can be retrieved by measuring the energy correlations $\sigma_{FW1,FW2}$, $\sigma_{FW1,SH2}$, $\sigma_{FW2,SH1}$, and $\sigma_{SH1,SH2}$ and solving a system of four coupled equations.

The last term on the right-hand side measures the correlation between the time delay between the two lasers and their parameters. This term can be retrieved after measuring $\sigma_{SF,FW1}$, $\sigma_{SF,FW2}$, $\sigma_{SF,SH1}$, and $\sigma_{SF,SH2}$ and solving a system of four coupled equation. At this point all terms on the right-hand side are known, and
Figure 9: Measured fluctuations of the arrival time between two synchronized lasers. $\Gamma_{T}^2 \sigma_T^2$ is plotted as function of the normalized time delay $T$ and the data points are fitted to a second-order polynomial function.

we can retrieve $\Gamma_{T}^2 \sigma_T^2$. Since $\Gamma_{T}^2 = 4T^2$ and $\sigma_T$ is independent of the normalized delay $T$, we expect $\Gamma_{T}^2 \sigma_T^2$ to be a quadratic function of the normalized time delay $T$. By a fit to the measured data we can extract $\sigma_T^2$ and we obtain $\sigma_T = 0.883$. From this we calculate $\Delta t^* = 25.7$ ps (with $\Delta T = \Delta t^*/\tau_1 + \tau_2^2$ and $\tau_1 = 16.5$ ps and $\tau_2 = 24$ ps corresponding to FWHM$_1 = 27$ ps, and FWHM$_2 = 39$ ps). As a result, the arrival time jitter between the two lasers amounts to about 50% of the cross-correlation width.

5. CONCLUSIONS

In conclusion we have demonstrated high fidelity all-fiber amplification of three different gain-switched laser diodes operating at 1064 nm, 1540 nm, and 1550 nm at 1 MHz repetition rate. In particular, the laser system at 1550 nm has been optimized such that it produces almost transform-limited pulses of 40 ps duration with a pulse energy of 0.485 $\mu$J. We have studied the statistics of all three laser systems individually and pairwise. The first study reveals that the pulses undergo fluctuations in pulse duration of the order of a few picoseconds. The latter study reveals that the arrival time jitter between the two synchronized laser systems is of the order of 25 ps, which corresponds roughly to half the pulse duration. Although this study shows the feasibility of frequency-mixing experiments using picosecond gain-switched laser diodes, we believe that the results can be improved by a further optimization of the electronic synchronization scheme.

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REFERENCES